The CQM model

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Abstract

I review a Constituent-Quark-Meson model (CQM) for heavy meson decays, outlining its characteristics and the calculation techniques developed for it. The strength of this effective model, is that it enables to evaluate heavy meson decay amplitudes through diagrams where the heavy mesons are attached at the ends of loops containing heavy and light quark internal lines. The phenomenological applications are presented in detail, trying to give a self-contained operative picture of the model.

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1 Introduction

During recent years, heavy meson physics has received a wide attention both from theory and experiment. This is because it helps the comprehension of many open problems of the standard model and can also act as a passage in the domain of new physics. Many experiments on B physics already at work or near to be started, BaBar, Belle, CLEO III, Hera-B, CDF-D0 and those planned to begin after 2005, ATLAS, CMS, LHCB and BTeV confirm this interest [1]. B physics has had an important role also in LEP I that has registered about 10^6 $Z^0 \to \bar{b}b$ events [2]. B decays offer the framework for investigating in detail the field of CP violations and for determining CKM (Cabibbo-Kobayashi-Maskawa) matrix elements. In particular, rare B decays, those in which there is no charm in the final state, are relevant for the research of signals of new physics [3]. In fact, the Standard Model predicts that rare B decays (the Cabibbo suppressed or the penguin induced decays) should be strongly suppressed, therefore, any anomalous increasing of branching ratios could be due, for example, to the existence of new particles, external to the standard model spectrum because interacting at higher energy scales.

The amplitudes governing heavy meson decays are theoretically calculated mainly using lattice QCD methods and the SVZ (Shifman-Vainshtein-Zakharov) sum rules [4].

The lattice QCD program [5], is that of computing the QCD partition functional summing over a representative ensamble of gauge fields and fermionic field configurations; the action is written in discrete form modelling the entire space-time as a four-dimensional grid where the distance between nearest neighboring sites is a and the linear dimension is $L \simeq \Omega^{\frac{1}{4}}$, Ω being the four volume of the grid. In principle, considering a sufficiently large number of configurations and simulating a very close ($a \to 0$) and large ($L \to \infty$) grid on a calculator, amounts to build a calculation framework nearly resembling that of continuous QCD. In practice, there are many technical problems: some of them have to do with computer power, some with the continuous limit of the results obtained on a discrete space-time grid.

In the ordinary hadronic matter, the quarks are not very far from each other, therefore, in ordinary circumstances, it is not essential to consider the complex QCD dynamics giving rise to the Abrikosov chromoelectric flux tubes thought to be responsible for quark confinement. In this situation valence quarks are weakly interacting with QCD vacuum fluctuations. The SVZ method aims at determining the parameters and the regularity of ordinary mesons and baryons through an expansion of the correlation functions, written in terms of dispersion intergrals, in a power series controlled by the α_s parameter (the strong coupling constant), plus power corrections expressed through the vacuum condensates ($G^2_{\mu\nu}$, $\bar{q}q$, $\bar{q}\sigma Gq, ..$). It is believed that the vacuum condensates contain the most relevant non perturbative effects of the QCD vacuum. Invoking the concept of *parton-hadron duality*, this expansion must be compared to the phenomenological expressions for the correlation functions. It is this comparison that allows to extract quantitative information on $2, 3, ..$-points correlators, *i.e.*, on all possible observables. One of the main drawbacks of SVZ sum rules is the difficulty one meets in computing the theoretical error due to the ambiguous choice concerning the truncation point of the series expansion.

This work is devoted to introduce an effective Constituent-Quark-Meson model based on a Lagrangian incorporating the symmetries of heavy quark effective theory, the chiral symmetry in the light quark sector, see section **2**, and, as is discussed in section **3**, dynamical information derived from an underlying Nambu-Jona-Lasinio interaction. In section **4**, together with the discussion of calculation techniques used for computing some relevant loop-integrals, it is shown how the determination of strong coupling constants, parameterizing the low energy effective hadron Lagrangian, proceeds through a comparison of the low energy matrix elements with the

CQM computed amplitudes: CQM plays the role of a fundamental model (since it contains, besides meson fields, also the elementary heavy and light quark fields) with which the hadron theory must match at higher energy, see discussion in section **2.1.1**. With respect to lattice QCD and SVZ sum rules, CQM is a rough approach that, anyway, has shown to be a quite reliable and easy-to-use method.

One of the very common problems of quark models [6], is that of associating theoretical errors to predictions. This topic is discussed for CQM in section **3**, together with the problem of defining the light constituent quark mass. The constituent quark mass is typically heavier than the current mass, appearing in the QCD Lagrangian (and related to the Higgs field VEV): one can think of a constituent quark as of a current (bare) quark dressed by a cloud of virtual particles generated by strong interactions [7]. The mechanism dressing the bare quark and giving the constituent quark its mass value, is an intrinsic feature of the model itself.

Section **5** is devoted to the study of exclusive semileptonic decays of B mesons through the CQM model. Here are examined processes involving $b \to c\ell\nu$ and $b \to u\ell\nu$ transitions, the former being related to V_{cb}, the latter to V_{ub}. In particular, CQM has allowed to obtain a prediction for the branching ratio of the semileptonic process $B \to a_1$.

All existing evaluations of exclusive semileptonic B decays are strongly model-dependent or are affected by problems related to the estimation of the theoretical error. Anyway an agreement among diverse models, *e.g.*, on the determination of a particular form factor, gives rise to a theoretical platform useful for a comparison with experimental data. This could also be an alternative approach to the study of rare B decays, considering that the most commonly used method to extract V_{ub} through a comparison with data, is the so called end-point-method, see, *e.g.*, [8]. The idea of the end-point-method is that of eliminating the background due to $b \to c\ell\bar{\nu}$ decays while examining the inclusive leptonic spectrum $\frac{d}{dE_\ell}\Gamma(b \to u\ell\nu)$ in the E_ℓ region where the invariant mass M_X of the hadron system emerging from the decay is such to avoid decays in a charmed final state: $M_X \leq M_D$. But, in this region of the energy spectrum, one meets technical difficulties related to the Wilson expansion of $\frac{d\Gamma}{dE_\ell}$: one can only compute the first terms of this expansion. Higher order terms depend on matrix elements of local operators having higher dimensionality, and can at most be estimated by phenomenological models. It is possible to show that, in the proximity of the end-point, *i.e.*, in the proximity of a certain critical value $M_{X,c}$, all terms in the Wilson expansion are equally important and, for even higher values of E_ℓ, the decay cannot anymore be analyzed by Operator-Product-Expansion. In the experiments devoted to the determination of V_{ub}, a kinematic cut on M_X, very near to the critical value $M_{X,c}$, is used. This means that the determination of V_{ub} is model-dependent since it is necessary to be able to estimate the terms having higher dimensionality in the Wilson expansion. To avoid this problem, one could think of enlarging the E_ℓ region experimentally examined. This could give the possibility of being far from $M_{X,c}$, but the price to pay is that of a strong growth of the background of events containing charm in the final state.

CQM gives the possibility of further investigating the exclusive channels $B \to \rho$, $B \to a_1$ and $B \to \pi$ in such a way to enlarge and give more solidity to the platform of model-dependent results I mentioned before.

2 Introduction to the formalism

2.1 Effective theories

In this section I will discuss briefly the general topic of effective theories in particle physics with the aim of introducing the basic ideas and tools of the CQM model in the subsequent sections.

When one calculates the energy levels for an hydrogen atom, the problem to face is that of solving the Schrödinger equation for an electron moving in the Coulomb potential generated by the positive proton charge: it is not relevant to take account of the inner quark structure of the proton. The low energy dynamics of the hydrogen atom does not depend in any relevant way on the high energy finer details of the proton inner structure. The proton can be simply considered as the static source of Coulombic potential and, in a first approximation, we can ignore also its spin and magnetic moment. Doing in such a way, the problem of determining hydrogen energy levels presents essentially only one energy scale m_e (the electron mass) and the dimensionless fine structure constant α: we have separated out higher energy scales. This can be done essentially because of the large separation of the energy scales that usually enter into a physical problem. A physical system in which there are different but close to each other energy scales, cannot be treated in the same way because even small perturbations can allow the system to explore all these scales with similar probabilities.

A finer calculation of the hydrogen energy levels requires to include in the calculation the effect of the spin and of the magnetic moment of the proton. These details are responsible of the well known hyperfine structure of the energy levels. We can state that the energy levels of the hydrogen atom can be computed ignoring the dynamics acting at scales larger than Λ, with $\Lambda >> m_e\alpha$, with an error of order $m_e\alpha/\Lambda$. The more the desired precision, the higher is Λ, the smaller is the error one makes ignoring the high energy ($> \Lambda$) dynamics. For example, parity violation effects at the atomic level are very small since the weak interaction energy scale is M_W, extremely larger than the atomic energy scale.

Effective theories [9]-[14] are those *models* conceived to describe the physics of a certain system at the energy scale of the experiment through which one studies it, *i.e.*, at the level of accuracy chosen to experimentally examine the system. In this sense, the atomic physics of the hydrogen atom is an effective theory of the hydrogen.

Effective models succeed in giving reliable phenomenological predictions where fundamental theories have many more technical and sometimes principle problems. Quantum-Chromo-Dynamics (QCD) is the most important example of a fundamental theory, *i.e.*, a theory derived from first principles, describing the intimate nature of strong interactions and the building fields of matter, that has deep troubles in dialing with the low energy hadron world. This is due to the still partial theoretical comprehension of the confinement mechanism of quarks in the hadronic matter. Therefore, to deal with hadrons, it is necessary to implement some low energy model, effective in the energy regions where the hadronic processes one wants to study take place.

A low energy effective theory of hadrons is anyway a relative of QCD, since it incorporates the symmetry properties required by the fundamental theory. The hadron effective Lagrangian must therefore be Lorentz invariant, the S-matrix must be unitary, the $\mathcal{PCT}$ symmetry must be obeyed and it has to show chiral symmetry in the limit in which light current masses are sent to zero. New symmetry properties could also emerge in the effective theory being absent in the fundamental one: the example relevant for this work is that of Heavy-Quark-Effective-Theory (HQET), to which is devoted the next section.

Symmetry properties select an infinite class of Lagrangian interaction terms, only a finite number of them being renormalizable. The requisite of renormalizability, crucial for a fundamental theory, is lost in the effective theory approach.

The origin of non-renormalizable interactions is due to the absence of heavy particles from the spectrum of the effective theory. An example comes from Fermi's β-decay theory, where a non-renormalizable four fermion contact term, distorts the high energy interaction mediated by the W particle, absent in Fermi's theory. Anyway Fermi's theory works extremely well at the energy scales of nuclear processes. The masses M of the heavy particles, excluded by the effective theory spectrum, may appear as energy cutoffs $\Lambda = M$ suppressing the non renormalizable terms by factors of E/M, E being the characteristic low energy scale of the processes described by the effective theory. For example, the typical energy scale of Quantum-Electrodynamics (QED) processes is of order of m_e, that is a sufficiently small number to explain way QED can be very well considered as a fundamental, renormalizable theory of electrodynamic interactions.

In general terms one can associate to each mass of a known particle a boundary between two different effective theories: the anomalous breaking of scale invariance, manifested in the peculiar distribution of particle mass values, gives then rise to a tower of separate effective theories. For energies below a certain boundary value, one can construct a low energy effective theory in which all the particle states above the boundary threshold are excluded from the spectrum. Of course, the coupling constants in the interaction terms related to the light fields should vary with continuity at the boundaries.

As we go down in the energy ladder, we meet effective theories containing less fields and a larger number of non-renormalizable terms while, in the opposite direction, we find that the non-renormalizable terms are progressively more important (less suppressed by E/M) and disappear at boundaries, where they are substituted by new renormalizable interaction terms. The important point is that *what happens at high energies doesn't affect the low energy behaviour.* This picture is deeply explained in [12], [13].

The renormalization group method [15] allows to bridge between two effective theories. The aim is that of calculating the low energy parameters through the high energy ones. These calculations can be explicitly performed only once the high energy theory is weakly coupled. The QCD case is therefore complicated because the renormalization group method doesn't allow to bridge continuously from the fundamental theory, the QCD, to the hadron effective theories. This is why, many times, the hadron low energy effective theory parameters are determined by a matching with some other more fundamental model, *i.e.*, some model containing in its spectrum also the higher energy elementary particles. These models are not necessarily QCD derived, like lattice-QCD or SVZ sum rules. In many cases these models contain hypotheses in conflict with the QCD structure. Object of this work is to introduce one of these effective models.

What is important to focus on, is that the proliferation of non-renormalizable terms (the irrelevant terms in the modern language), doesn't spoil the predictive power of the effective theory. On the contrary, non-renormalizable terms can help in determining the predictive power at disposal.

Here follows an example of how the effective theory approach could make things very easy with respect to a fundamental theory approach.

2.1.1 Photon-photon scattering

Let us suppose to be interested in understanding how the cross section for the photon-photon scattering scales with the energy of the photon in the limit in which this is lower than the rest energy of the electron. From an effective field theory point of view, this means that we are interested in building an effective theory in which the electron is excluded by the particle spectrum. The electron mass acts as the cutoff $\Lambda = m_e$ discussed before.

We therefore only need an interaction Lagrangian containing four photon fields. The symmetry principles instructing us about how to build this low energy effective theory are: Lorentz invariance, gauge invariance and the $\mathcal{P}, \mathcal{C}, \mathcal{T}$ symmetries. To the lowest order we can therefore